PARKER

Self Induction Formulas

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SELF INDUCTION FORMULAS

BY

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THIS IS TO CERTIFY THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

WASHINGTON WARNER PARKER

ENTITLED   SELF-INDUCTION FORMULAS

IS APPROVED BY ME AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF Bachelor of Science in Electrical Engineering

Morgan Brooks,
Instructor in Charge.

APPROVED: *Morgan Brooks.*

HEAD OF DEPARTMENT OF Electrical Engineering

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Self Induction Formulas.

SELF INDUCTANCE FORMULAS.

Introduction.

The purpose of this paper is to present and discuss a few of the best known formulas for the inductance of any coil without an iron core, and to present and discuss a new formula, developed by Professor Brooks of the University of Illinois. This new formula is simple, easily used, and is accurate within about three percent for all coils.

The following notation will be used in presenting all the formulas. The notation of the authors will be changed to correspond.

n = total number of turns.

n_i = number of turns per unit length.

r = mean radius.

D = external diameter.

t = radial thickness.

l = length of coil.

p = radius of section of winding, when round.

R = geometric mean distance of section of coil.

Fig. I.

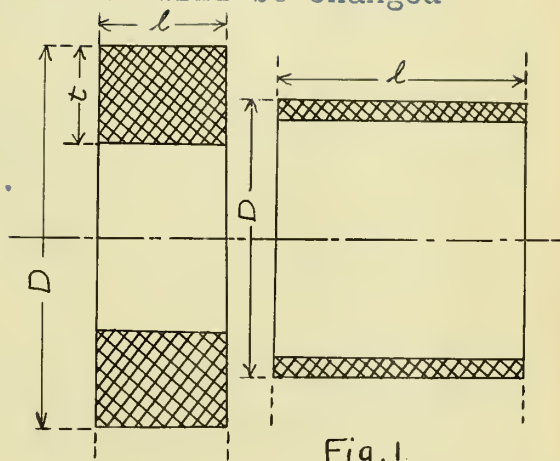The first formulas we shall present are a few of those developed by Maxwell.

Maxwell's Formulas.

The simplest formula by Maxwell was derived as follows: The field intensity in the coil, due to the current flowing, is $H = \dfrac{4 \pi n i}{l}$ where i is the current, n the number of turns, and l the length of the coil in centimeters.

The area of the cross-section of the coil is πr^2 so that

the flux through the coil is,

$$\phi = \frac{4\pi^2 r^2 n i}{l}$$

Since there are n turns in the coil, the flux turns are

$$\phi_t = \frac{4\pi^2 r^2 n^2 i}{l}$$

The inductance L, is equal to the rate of change of flux with respect to the current, hence,

$$L = \frac{d\phi_t}{i} = \frac{4\pi^2 r^2 n^2}{l} \quad \text{---------} \quad (1.)$$

This formula (1) is accurate only for extremely long solenoids.

Another formula developed by Maxwell is,

$$L = \frac{4}{3}\pi^2 n_1^4 l \, (x-y)(x^3-y^3) \quad \text{--------------} \quad (2)$$

In this formula, x and y are the external and internal radii of the coil. The use of this formula may lead to errors varying from twelve to thirty-five percent, for even comparitively long coils.

Maxwell has left us several very accurate formulas for the self inductance of circles, consisting of one turn of wire. They are as follows:

$$L = 4\pi r \left\{ \left(\frac{1+3R}{16 r^2}\right) log_\epsilon \frac{8r}{R} - \left(2 + \frac{R^2}{16r^2}\right)\right\} \quad \text{--------} \quad (3)$$

$$L = 4\pi r \left\{ \left(1 + .1137\frac{\rho^2}{r^2}\right) log_\epsilon \frac{8r}{\rho} - .0095 \frac{\rho^2}{r} - 1.75\right\} \quad \text{------} \quad (4)$$

$$L = 4\pi r \left(log_\epsilon \frac{8r}{R} - 2 \right) \quad \text{----------------------} \quad (5)$$

Formula (4) is for a coil of circular cross-section, and is obtained from (3) by substituting for R, its value $\rho\epsilon^{-\frac{1}{4}}$ It is very accurate for circles where $\frac{\rho}{r}$ is very small. Formula (5) is an approximate formula, and is derived from (3) by dropping some of the terms.

Stefan's Formulas.

Stefan derived a very accurate formula for short coils. It is as follows,

$$L = 4\pi n^2 r \left\{ \log_\epsilon \frac{8r}{\sqrt{l^2+t^2}} \left(1 + \frac{3l^2+t^2}{96r^2}\right) - y_1 + \frac{l^2}{16r^2} y_2 \right\} \quad \text{------------} \quad (6)$$

In which y_1 and y_2 are constants depending on the ratio of $\frac{l}{t}$ or $\frac{t}{l}$. A table giving these constants may be found in the back of this paper.

In order to correct for the insulation, add the term

$$\Delta L = \left\{ 4\pi r n \left(\log_\epsilon \frac{m}{n_o} + .15494\right) \right\}. \qquad \text{in which m is}$$

the diameter of the insulated wire, and n_o is the diameter of the bare wire.

Other Formulas for the Self Inductance of Circles.

$$L = 4\pi r \left\{ \log_\epsilon \frac{4\pi r}{\rho} - 1.508 \right\} \text{------------------------} (7)$$

(7) is nearly equivalent to,

$$L = 4\pi r \left\{ \log_\epsilon \frac{8r}{\rho} - 1.75 \right\} \text{----------------------} (8.)$$

These formulas are given by Kirchhoff, and are approximate formulas.

$$L = 4\pi r \left\{ \left(1 + \frac{\rho}{2r} - \frac{\rho^2}{32r^2}\right) \log_\epsilon \frac{8r}{\rho} - \left(2 + \frac{5\rho}{8r} + \frac{19\rho^2}{16r^2}\right) \right\} \text{---------} (9.)$$

If the wire is small, this reduces to approximately

$$L = 4\pi r \left(\log_\epsilon \frac{8r}{\rho} - 2\right) \text{--------------------------} (10)$$

(9) and (10) are the formulas dericed by Minchin. They are inaccurate.

Probably the most accurate formula derived for the Self Inductance of Circles, is that derived by Max Wein. If the ring, figure 2 of radius r and radius of section ρ is considered to be made up of an infinite number of circular filaments, the self inductance of the ring is equal to the mean value of the sum of the

Fig 2.
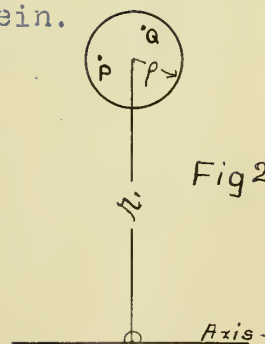
others. If, therefore, we express the mutual inductance of an
element at P and another at Q, and integrate over the entire
area of the section, we obtain the mutual inductance of the
filament P on the entire ring. Integrating again over the en-
tire section of the coil we obtain the self inductance of the
ring. Wein's result obtained in this way is as follows:

$$L = 4\pi\lambda\left\{\left(1+\frac{\rho^2}{8\lambda^2}\right)\log_\varepsilon\frac{8\lambda}{\rho} - .0083\frac{\rho^2}{\lambda^2} - 1.75\right\} \quad\text{----------------(11)}$$

This formula applies to a circle of one turn of wire.
If the current sheet is not uniformly distributed, and if the
intensity is assumed to be proportional to the distance from
the axis of the ring, the inductance is,

$$L = 4\pi\lambda\left\{\left(1+\frac{3}{8}\frac{\rho^2}{\lambda^2}\right)\log_\varepsilon\frac{8\lambda}{\rho} - .092\frac{\rho^2}{\lambda^2} - 1.75\right\} \quad\text{-----------(12)}$$

If the less important terms be dropped, these formulas
reduce to those of Kirchhoff and Maxwell.

Rayleigh's Formulas.

Rayleigh's formula for a narrow current sheet Fig. 3 is,

$$L = 4\pi\lambda n^2\left\{\log_\varepsilon\frac{8\lambda}{\ell} - .5 + \frac{\ell^2}{32\lambda^2}\left(\log_\varepsilon\frac{8\lambda}{\ell} + \frac{1}{4}\right)\right\} \quad\text{------------(13)}$$

Rayleigh and Niven are responsible for the
following formula for a coil of n turns of cir-
cular section.

$$L = 4\pi n^2\lambda\left\{\left(1+\frac{\rho^2}{8\lambda^2}\right)\log_\varepsilon\frac{8\lambda}{\rho} + \frac{\rho^2}{24\lambda^2} - 1.75\right\} \quad\text{-----(14)}$$

Fig. 3
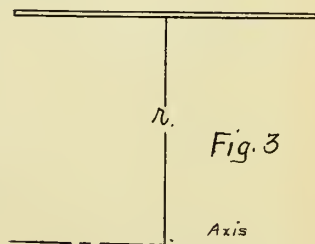
If the number of turns is greater than one, a correction
for insulation must be made.
Another formula practically the same is,

$$L = 4\pi n^2\lambda\left(\log_\varepsilon\frac{8\lambda}{\rho} + \frac{1}{3}\right)\frac{\rho^2}{8\lambda^2} + \log_\varepsilon\frac{8\lambda}{\rho} - \frac{7}{4} \quad\text{---------------(15)}$$

For spirals, in which the axial dimension may be considered zero,

$$L = 4\pi r n^2 \left\{ \log_e \frac{8r}{t} - \frac{1}{2} + \frac{t^2}{96 r^2} \left(\log_e \frac{8r}{t} + \frac{43}{12} \right) \right\} \text{---------} (16)$$

For long coils of one layer, of wire so small that "t" can be considered equal to zero,

$$L = 4\pi r n^2 \left\{ \log_e \frac{8r}{l} - \frac{1}{2} + \frac{l^2}{32 r^2} \left(\log_e \frac{8r}{l} + \frac{1}{4} \right) \right\} \text{---------} (17)$$

These formulas are quite accurate within their limits. Mr. Louis Cohen of the Bureau of Standards has derived the following exact formula for the self inductance of a single layer solenoid.

$$L = 4\pi n_1^2 \left\{ \frac{l^4 + 4r^2 l^2}{3\sqrt{4r^2 + l^2}} F + \frac{4r^2 - l^2}{3} \sqrt{4r^2 + l^2} E - \frac{8 r^3}{3} \right\} \text{-------} (18)$$

F and E are the complete elliptical integrals of the first and second kinds, to the modulus k, where $k = \frac{4r^2}{4r^2 + l^2}$

E and F can be put in the form of a series,

$$F = \frac{\pi}{2} \left\{ 1 + \frac{1}{2^2} k^2 + \frac{1^2 \times 3^2 \times k^4}{2^2 \times 4^2} + \cdots \right\}$$

$$E = \frac{\pi}{2} \left\{ 1 - \frac{1}{2^2} k^2 - \frac{1^2 \times 3^2 \times k^4}{2^2 \times 4^2} - \cdots \right\}$$

If l = 4r, E and F will reduce to approximately

$$F = \frac{\pi}{2} \left\{ \frac{5r^2 + l}{4r^2 + l} \right\} \text{--- --- --------} a.$$

$$E = \frac{\pi}{2} \left\{ \frac{3r^2 + l}{4r^2 + l} \right\} \text{--- --------} b.$$

Substituting in formula 18,

$$L = 4\pi^2 n_1^2 \left\{ \frac{2r^4 + r^2 l^2}{\sqrt{4r^2 + l^2}} - \frac{8 r^3}{3 \pi} \right\} \text{--------------} (19)$$

Form (19) is much simpler and easier to use.

For a coil of more than one layer, Mr. Cohen has developed the following formula, in which m is the number of layers.

$$L = 4\pi^2 n_1^2 m \left\{ \frac{2r_0^4 + r_0^2 l^2}{\sqrt{4r_0^2 + l^2}} - \frac{8r^3}{3\pi} \right\} + 8\pi^2 n_1^2 \left[\left[(m-1)r_1^2 + (m-2)r_2^2 + \cdots \right] \right.$$

$$\left(\sqrt{r^2+l^2} - \frac{7}{8}r_1 \right) + \frac{1}{2}\left[m(m-1)(m-2)r_2^2 + \cdots \right]$$

$$\left(\frac{r_1 \delta r}{\sqrt{r_1^2+l^2}} - \delta r \right) - \frac{1}{2}\left[m(m-1)r_1^2 + (m-2)(m-3)r_3^2 + \cdots \right]\frac{\delta r}{8} \right\} \quad -----(20)$$

For most coils, the last term of formula (20) is so small
as to be entirely negligable. For coils where the length is
four times the diameter, the last two terms disappear, and the
formula becomes,

$$L = 4\pi^2 n_1^2 m \left\{ \frac{2r_0^4 + r_0^2 l^2}{\sqrt{4r_0^2 + l^2}} - \frac{8r_0^3}{3\pi} \right\} + 8\pi^2 n_1^2 \left[\left[(m-1)r_1^2 + (m-3)r_2^2 + \cdots \right] \right.$$
$$\left. \left(\sqrt{r_1^2+l^2} - \frac{7}{8}r_1^3 \right) \right\} \quad -----------(21)$$

The subscripts refer to the different layers. The longer
the coil the more exact this formula becomes. It becomes more
difficult to handle, as the number of layers is increased.

Another formula developed at the Bureau of Standards,
and especially applicable to short coils, is:

$$L = 4\pi r n^2 \left\{ \begin{array}{l} \left(\log_\epsilon \frac{8r}{l} - \frac{1}{2} \right) + \frac{l^2}{32r^2}\left(\log_\epsilon \frac{8r}{l} + \frac{1}{4} \right) \\[2mm] - \frac{1}{1024}\frac{l^4}{r^4}\left(\log_\epsilon \frac{8r}{l} - \frac{2}{3} \right) \\[2mm] + \frac{10}{131072}\frac{l^6}{r^6}\left(\log_\epsilon \frac{8r}{l} - \frac{109}{120} \right) \\[2mm] - \frac{35}{4194304}\frac{l^8}{r^8}\left(\log_\epsilon \frac{8r}{l} - \frac{431}{420} \right) + \cdots \end{array} \right. \quad --------(22)$$

It is desired, before proceeding further, to call attention
to the fact that all these formulas, except several due to
Maxwell, give exact results when the dimensions of the coil are
given exactly. Few engineers have at hand the necessary ap-
paratus for an absolutely accurate determination of dimensions.
Furthermore very few of these formulas could be used by en-
gineers, because they are too complicated and require too

much time. Moreover, there could be no object in using a complicated formula if a simpler one that is nearly as accurate could be used, especially if it is considered that the dimensions can not be measured accurately enough to warrant using an exact formula.

Professor Brook's Formula.

The formula that we are about to present, possesses the merits of simplicity and fair accuracy. It is one that can be readily solved on a slide rule. It is, of course, purely empirical.

$$L = \frac{10(D-t)^2 n^2}{l + t + .5D} \times \frac{10l + 6t + D}{10l + 5t + .7D} \times \log_{10}\left(10 + \frac{D}{10l + 15t}\right) \ -\ -\ -\ -\ -\ A$$

For coils in which the length is three or four times the diameter, the logarithmetic term disappears. In the case of still longer coils, the second term becomes negligible, and

$$L = \frac{10(D-t)^2 n^2}{l}, \qquad \frac{D-t}{2} \text{ is the mean radius, and 10 is}$$

$1.012 \pi^2$ hence the formula becomes approximately, $L = \frac{4\pi^2 r^2 n^2}{l}$, which is identical with the first formula developed by Maxwell. (Formula 1).

The logarithmetic term can be neglected for practically all coils. It becomes of importance only in the case of rings, when D becomes large with respect to l and t.

Now let us compare the results obtained by the use of this formula, with some of those just given.

Let us take a circle of the following dimensions:

N = 25,   n = 1,   t = 1,   = 1,   D-t = 50.

By Weinstein's and Stefan's formulas, L = 1290 cm.

By Maxwell's, using R,   L = 1289 cm.

By Brooks' formula   L = 1271 Cm.

The error is about 1.6%.

Let us try another ring in which l and t are both one-tenth of a centimeter, and in which r = 25 and n = 1.

By Weinstein's L = 2015 cm.

By Stefan's   L = 2015 cm.

By Maxwell's   L = 2015 cm.

By Brooks   L = 2062 cm.

Error = 2.3%.

Suppose we try a coil. Let,

2 r = D-t = 50,   = 2, t =.1, n = 20. This form of winding gives what could be called a narrow current sheet.

By Rayleigh's formula for a narrow current sheet,

L = 163,558   cm. = 513,574 cm. = .000513 henrys.

By Stefans, L = 163,559   cm. = .000513 henrys.

By Brooks' L = .000511 henrys.

The error is about .4 percent.

Let us try a coil in which l is greater and d is less.

$2 r = D-t = 10 cm.$ $l = 10 cm.$ $t = 1 cm.$ $n = 10$

By Lorenz's formula, for one layer,

$$L = \frac{4\pi n^2}{3 l^2}\left\{d\left(4r^2-l^2\right)E + d\,l^2 F - 8 r^3\right\}$$

where $d = \sqrt{4r^2+l^2}$ and E and F are the complete eliptical integrals to modulus k, where $k = \frac{2r}{d} = \frac{2r}{\sqrt{4r^2+l^2}}$.

The correction term is, $\Delta L = 4\pi r n\left(A+B\right)$,   when A and B are constants, and are 0.6922 and .2792 respectively.

L = 1968.45   cm. = 6181 cm.

By Brooks formula, L = 6320 cm.

The error is about 2.2%.

The same degree of accuracy was found for all other coils given by the Bureau of Standards.

A standard coil has been recently constructed by the Bureau of Standards at Washington, D.C. In the table appearing below may be found the dimensions of the coil, the inductance, observed by the Bureau of Standards, the value of inductance calculated by Professor Brooks formula, and the percent error.

Table 1.

Showing the Dimensions of the Several Sections of the Bureau of Standards Inductance Standard.

Coil.	Number of turns	Length in Centimeters	Diameter in Centimeters	Observed Inductance Henry's	Inductance Calculated by Prof. Brooks Fm'la	% error
1	221	15.2928	54.1724	.0362	.0371	2.4%
2	251	17.3140	54.1724	.0442	.0445	.7
3	189	13.1520	54.1724	.0283	.0283	0
1+2+3	661	45.8432	54.1724	.180	.1809	.4
1 2	472	32.6487	54.1724	.113	.1142	1.
1 3	440	30.5085	54.1724	.102	.1029	.9

A Standard Inductance Coil has also been built by the Clark University people. Following, is a table showing the dimensions, etc.

$$L_{Max} = \frac{.343 \, C^2}{l}, \text{ where } C^2 = 4\pi^2 \lambda^2 n^2.$$

L is in centimeters. To reduce it to henrys, divide by 10^9
The volume of a coil of wire is,

$$V = 2\pi \left(\frac{D-t}{2}\right) l \cdot t$$

$\frac{D-t}{2}$ is the mean radius. Substituting the ratios for
maximum inductance,

$$V = \pi \, (3t) \times \frac{4}{3} t \times t = 4\pi t^3$$

$$t = \sqrt[3]{.0795V} \quad\text{---------------------------} (E)$$

This relation holds, whether the measurements are made in
in inches or centimeters. If the wire is insulated a factor
must be placed under the radical sign. f is the percent of
copper in any volume of winding. Values of f may be found in
Table 3, for various sizes of wire.

$$t = \sqrt[3]{\frac{.0795 \, V}{f}} \quad\text{-------------------} (E')$$

Formula D, L max = $\frac{.343 \, C^2}{l}$ reduces to,

$$\text{L max} = \frac{125 F^2}{l_1} = \frac{93.75 \, F^2}{t} \quad\text{-----------------} (F)$$

where F is the length of wire in feet, and l_1 is the length of
the coil in inches.

Having the two formulas (E) and (F), it is easy to determine
the maximum inductance that can be obtained from any length of
wire. For example, let us take 1000 feet of number 16 wire,
and determine the form of the coil giving maximum inductance.
1000 feet of number 16 wire has a volume of 24.3 cubic inches.
The copper factor, from Table 3, is .64. Hence,

$$t = \sqrt[3]{\frac{.0795 \times 24.3}{.64}} = 1.44 \text{ inches}.$$

$$l = \frac{4}{3} t = 1.92 \text{ inches}, \quad D = 5.76 \text{ inches}.$$

$$L_{Max} = \frac{125 \, (1000)^2}{10^9 \times 1.92} = .0651 \text{ henrys}.$$

The same results are obtained by using the metric system. The value of L is given in centimeters by the formula, and must be divided by 10 to be reduced to henrys.

Table 4 has been constructed by the use of this formula. It gives the length of the coil to give maximum inductance. Knowing the other two dimensions can be determined from the equation,

The length of the coil has been determined for different lengths and different sizes of wire, as shown by the table.

Table 5 gives the maximum inductance obtainable from various lengths, and different sizes of wire.

Curves on Plate 1 show the relations expressed by Table 4, and the curves on Plate 2 show the relations expressed in Table 5.

Experimental Results.

Garrison and Williams constructed a set of coils for their Thesis work last year. The set consisted of ten coils each of about ten centimeters mean radius, two centimeters axial length, and two centimeters radial length. Each coil consisted of a hundred turns of double cotton covered number 19 wire. These coils could be placed in series one above another in such a way as to vary the length of the coil as desired. This coil was used in this Thesis. The data for this coil appears in Table 6, and the curves, showing the relation between the value calculated, and the value observed, appears on Plate 3.

A very interesting set of experiments was performed on an air coil constructed several years ago in the Electrical Laboratory. This coil consists of two parts, a primary coil can be slipped in and out of the secondary any desired distance.

It is balanced in any position by heavy counter weights. A
diagram of this piece of apparatus appears on Plate

The secondary coil consisted of 418 turns of number 8 wire,
wound in two layers. The external circumference of the winding
was 30.75 inches. The resistance was .677 ohms. The inductance
was .0119 henrys.

The primary coil consisted of 422 turns of number 8 wire,
wound in two layers. The external circumference was 26.2 inches.
The resistance was .576 ohms. The inductance was .009 henrys.
Both coils were 30 inches long.

Measurements were made as follows. The primary coil was
connected in series with the secondary, in such a manner as to
have the inductance of one coil opposing that of the other.
The zero position of the coils, was the position in which one
coil was entirely inside the other. As the primary coil was
gradually withdrawn, the inductance, measured by the impedance
method, increased until the coils were separated as far as the
construction of the apparatus would permit, ie. e. about four
inches. Next, the coils were connected in series so that the
inductance was in the same direction. In this case, the max-
imum inductance was at the zero position. As the coil was with-
drawn, the inductance decreased.

In the third experiment the inductance of the secondary
was measured with the primary short circuited. The inductance
increased gradually as the coil was drawn out.

The same experiment was made with the secondary coil short
circuited. Practically the same results were obtained.

These various results, and the curves for them may be
found in Table 7   and on Curve Sheet 4, respectively.

The results lead us to expect that if the inductance of the two coils had been equal, they would have exactly neutralized each other, when connected in opposition.

Summation.

We have seen that the formulas of Weinstein, Stefan, Maxwell and Kirchhoff are very accurate for ring and very short coils. We have found that the formulas developed by Mr. Louis Cohen and Professor Coffin are, some of them accurate for short coils and some for long coils, but all of them are complicated, and impossible of rapid solution. They all require logarithmetic talbes of hyperbolic logarithms. We have shown Professor Brooks' formula to be very easy of solution, and accurate within three percent for all coils. Our conclusion is that the formula last mentioned is accurate enough to warrent its use in all engineering work.

Table

Showing values of constants y_1 and y_2 for Stefan's formula for Inductance.

$\frac{l}{t}$ or $\frac{t}{l}$	y_1	y_2	$\frac{t}{l}$	y_1	y_2
0.00	0.5	0.1250	0.50	0.79600	0.3066
.05	.54899	.1269	.55	.80815	.3437
.10	.59243	.1325	.60	.81823	.3839
.15	.63102	.1418	.65	.82648	.4274
.20	.66520	.1548	.70	.83311	.4739
.25	.69532	.1714	.75	.83831	.5234
.30	.72172	.1916	.80	.84225	.5760
.35	.74469	.2152	.85	.84509	.6317
.40	.76454	.2423	.90	.84697	.6902
.45	.78155	.2728	.95	.84801	.7518
.50	.79600	.3066	1.00	.84834	.8162

Table No. 3.

B & S Gauge No.	Dia. in inches	Percent Copper in any Volume.	Volume of 1000 ft.of Copper-bare.	Volume of 1000 ft.of insulated copper.
2	.2576	.76	625	822
4	.2043	.75	395	526
6	.162	.74	248	335
8	.1285	.73	156	214
10	.1019	.72	97.86	136
12	.0808	.71	61.5	86.6
14	.06408	.70	38.7	55.3
15	.05707	.68	30.7	45.3
16	.05082	.64	24.3	38.0
17	.04526	.62	19.3	31.1
18	.04030	.61	15.3	25.1
19	.03539	.60	11.8	19.7
20	.03196	.58	9.6	16.5

Table 4.

Values of   in inches for coils wound with different size of
wire and different lengths of wire, in forms to give maximum
inductance.

	500	1000	1500	2000	2500	3000	3500	4000.
2		5.37	6.00	6.62	7.20	7.75	8.20	8.53
4		4.62	5.19	5.73	6.21	6.68	7.08	7.40
6		3.98	4.51	5.03	5.40	5.75	6.08	6.32
8		3.43	3.92	4.32	4.68	4.93	5.21	5.45
10	2.33	2.95	3.39	3.74	4.	4.27	4.47	4.68
12	2.01	2.53	2.91	3.20	3.44	3.80	3.86	4.03
14	1.73	2.18	2.49	2.75	2.96	3.20	3.31	3.47
15	1.61	2.04	2.33	2.57	2.77	2.93	3.10	3.24
16	1.53	1.92	2.20	2.43	2.61	2.77	2.92	3.06
17	1.43	1.80	2.06	2.26	2.45	2.60	2.74	2.86
18	1.32	1.68	1.92	2.10	2.28	2.41	2.54	2.66
19	1.22	1.55	1.77	1.95	2.09	2.22	2.34	2.45
20	1.17	1.46	1.67	1.84	1.96	2.11	2.21	2.31

To determine remaining dimensions:

Table No. 5.

Values of Inductance in Henrys for Different Sizes of Wire of
Different Lengths, Wound into Coil of Dimensions to give Max-
imum Inductance.

#B & S. Gauge.	500	1000	1500	2000	2500	3000	3500	4000
2		.0243	.045	.0736	.108	.145	.190	.234
4		.0270	.055	.0872	.125	.168	.220	.278
6		.0314	.061	.0994	.145	.196	.252	.317
8		.0364	.072	.1158	.168	.228	.295	.368
10	.0136	.0424	.0833	.134	.195	.263	.343	.427
12	.0155	.0494	.0977	.156	.227	.296	.397	.496
14	.0181	.0573	.113	.181	.264	.351	.463	.576
15	.0194	.0613	.120	.194	.282	.384	.494	.617
16	.0204	.0651	.128	.206	.300	.406	.525	.654
17	.0218	.0694	.136	.221	.319	.432	.560	.700
18	.0237	.0745	.141	.238	.343	.466	.603	.752
19	.0256	.0806	.158	.256	.374	.506	.655	.816
20	.0267	.0856	.168	.272	.396	.533	.692	.865

Table 6.

DATA.

Coils Constructed by Garrison and Williams.

Observed Values.

Coils	l	n	r	t	L	Computed L by Brooks Formula.	
					L	L	Error %
	18.9	900	10.3	2	.116	.111	−3
	16.8	800	"	2	.095	.0940	−1
	14.7	700	"	2	.0778	.0772	− .8
	12.6	600	"	2	.0500	.0616	+ 2.6
	10.5	500	"	2	.0450	.0448	+ .4
	8.4	400	"	2	.0335	.0338	+ .9
	6.3	300	"	2	.0219	.0216	− .9
	4.2	200	"	2	.0110	.01095	− .5
	2.1	100	"	2	.0034	.00334	− .5

Table 7.

Data for Slip Coil.

Coils connected	E	I	Observed. Position of coil-inches.	L.
in opposition.	17.5	9.56	0	.00292
	20.5	9.52	4	.00464
	26.	9.31	8	.00662
	32.5	9.15	12	.00882
	38.5	9.00	16	.01086
	46	8.62	20	.0137
	51.5	8.42	24	.0158
	55.5	8.08	28	.0179
	58.	7.88	32	.0192
Coil Connected	105.5	7.60	0	.0367
in Series with	105	7.76	4	.0358
Inductance	104	8.16	8	.0337
acting in same	102	8.63	12	.0311
direction.	100.5	9.12	16	.0289
	98	9.77	20	.0264
	91	9.90	24	.0242
	83	9.89	28	.0220
	79.5	10.1	32	.0207

Table 7.

Data for Slip Coil Continued.

	E	I	Position of coil-inches.	L.
Primary Coil	18.0	9.70	0	.00459
Short Circuited.	20.5	9.70	4	.00531
	24.2	9.71	8	.00637
	30.5	9.67	12	.00818
	34.5	9.62	16	.00935
	38.0	9.59	20	.01033
	40.	9.53	24	.01098
	41.5	9.46	28	.01115
	41.	9.44	32	.01138
	40	9.24	30	.01136
Secondary Coil	14	9.86	0	.00344
Short Circuited.	16	9.86	4	.00403
	20	9.87	8	.00516
	23.5	10.00	12	.00604
	26.5	10.00	16	.00685
	29.5	10.00	20	.00768
	31.	9.89	24	.00817
	32.	10.01	28	.00836
	32.	10.01	32	.00836

PLATE 1

CURVES GIVING LENGTH OF COILS WHEN WOUND FOR MAXIMUM VALUE OF INDUCTANCE, FOR DIFFERENT LENGTHS OF WIRE.

Length of Coil in Inches.

Length of Wire in Feet.

Nº 2 Wire - B & S Gauge
Nº 4
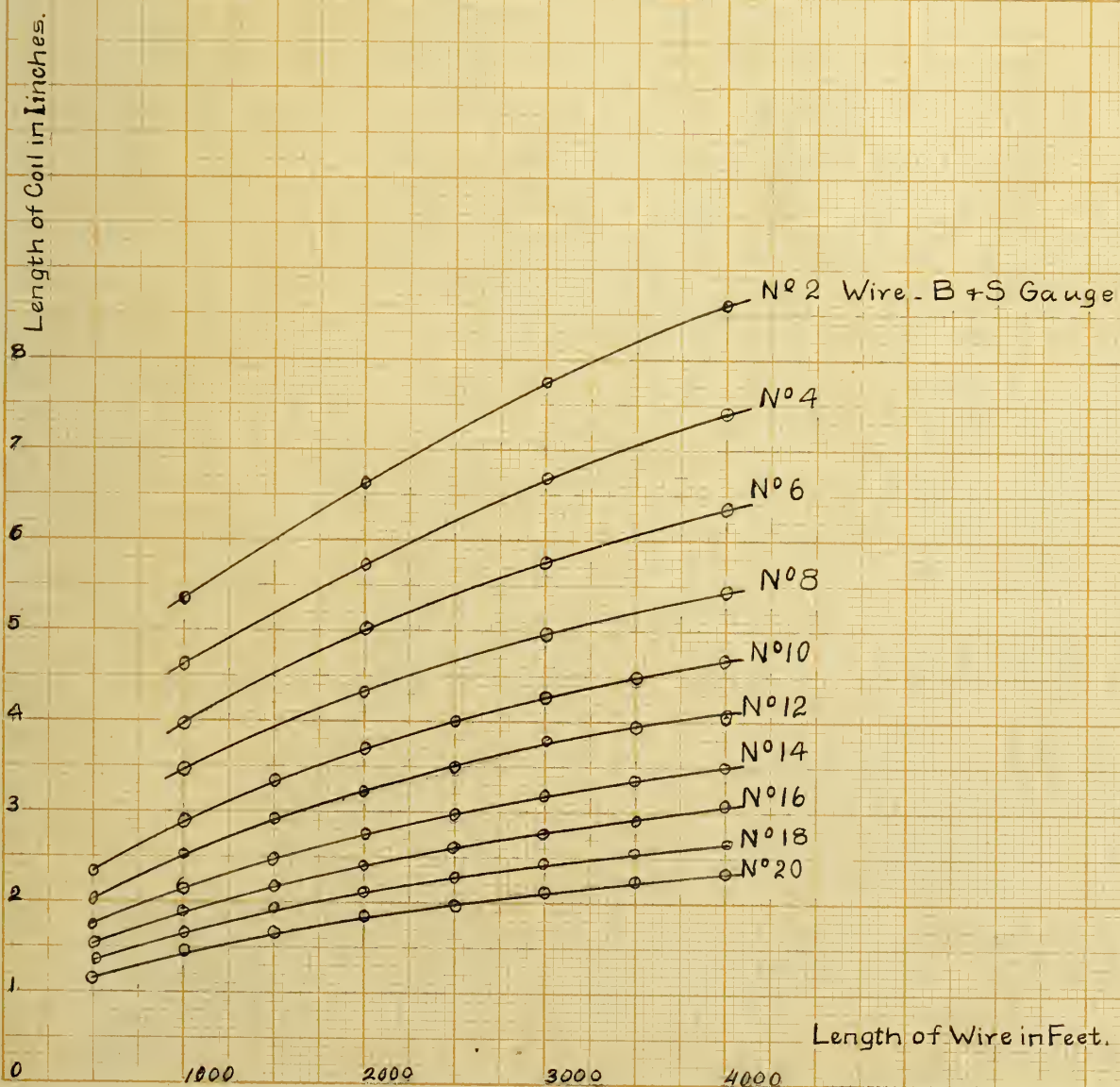Nº 6
Nº 8
Nº 10
Nº 12
Nº 14
Nº 16
Nº 18
Nº 20

PLATE 2.

CURVES SHOWING MAXIMUM
POSSIBLE INDUCTANCE
OBTAINABLE
FROM
VARIOUS LENGTHS OF WIRE.
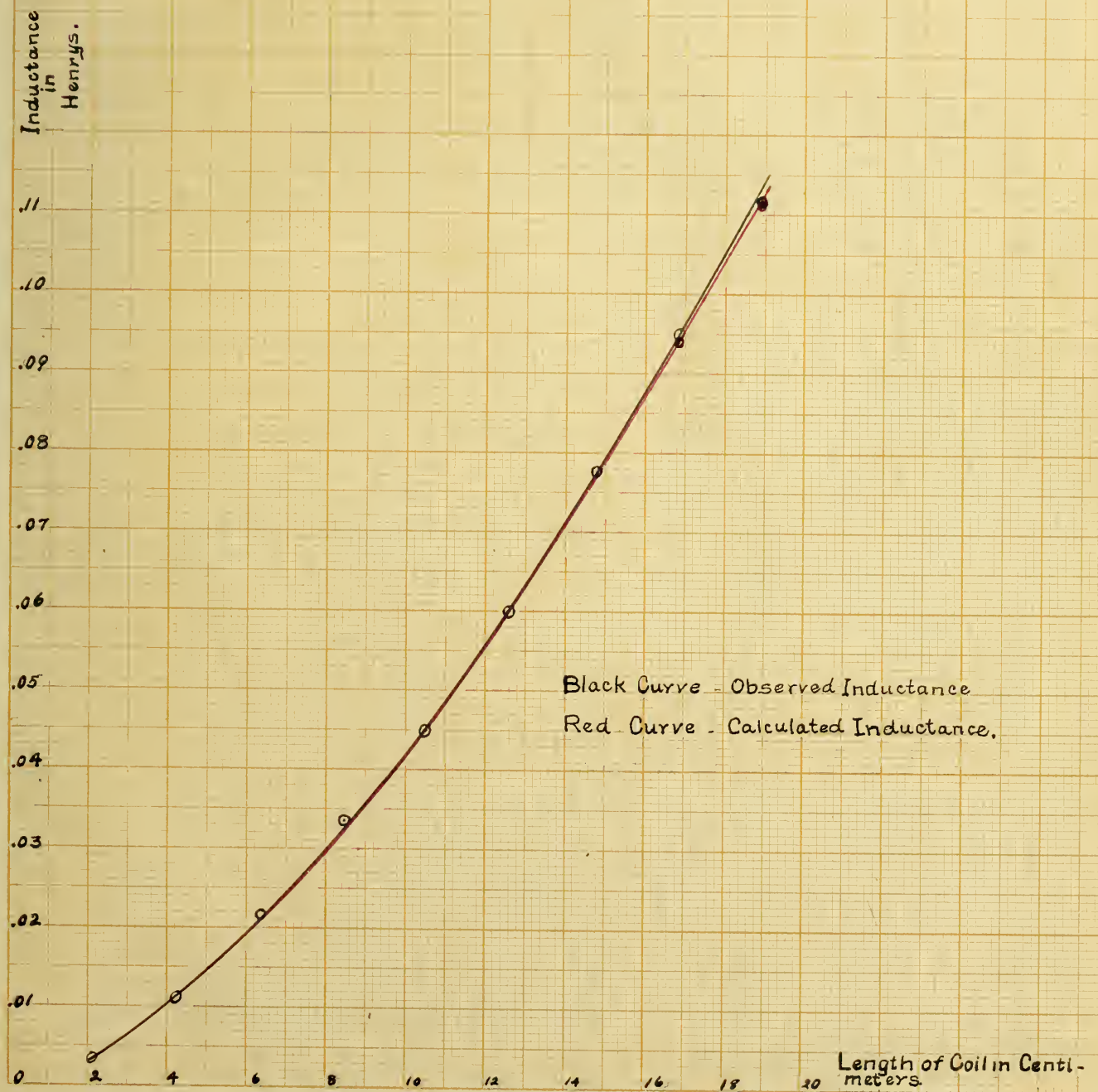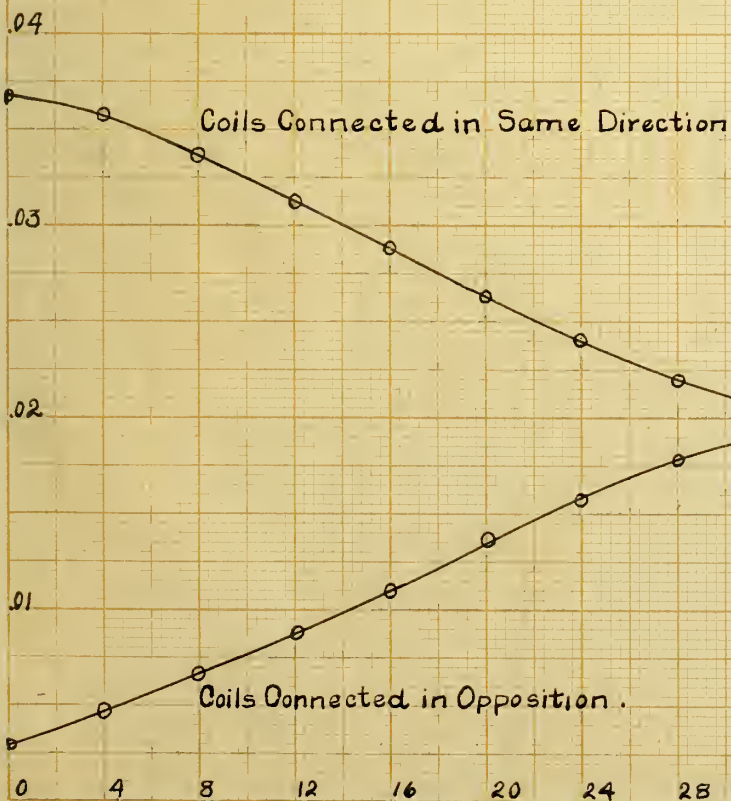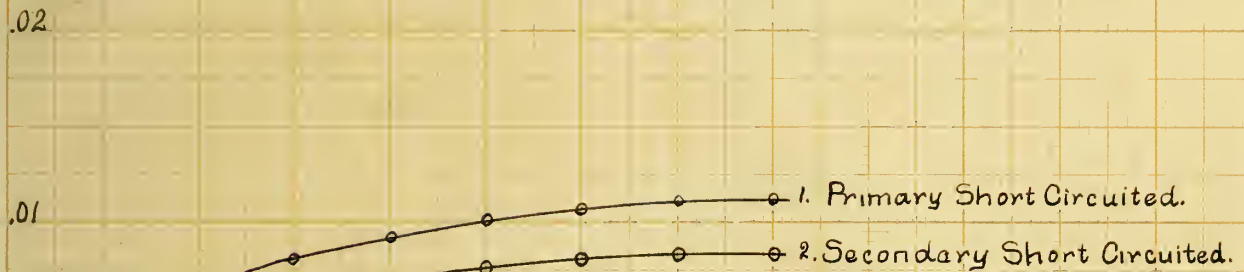
PLATE 3

COIL CONSTRUCTED
BY
GARRISON AND WILLIAMS.

Inductance in Henrys.

Black Curve - Observed Inductance
Red Curve - Calculated Inductance.

Length of Coil in Centimeters.

U OF I S S FORM 1

PLATE 4

CURVES
FOR
SLIP COIL.
University of Illinois

Inductance in Henrys

.02

.01 — 1. Primary Short Circuited.

2. Secondary Short Circuited.

0

.04

Coils Connected in Same Direction

.03

.02

.01

Coils Connected in Opposition.

0   4   8   12   16   20   24   28   32

Distance of Primary Coil in Inches
from its Zero Position.